Finite differences

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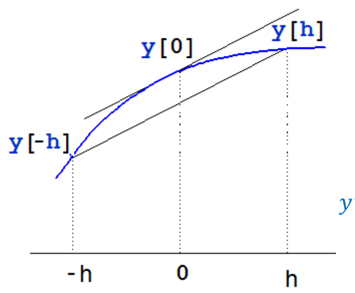
3-point FINITE DIFFERENCES



$$y = a + bx + cx^2$$
$$y' = b + 2cx$$
$$y'[0] = b$$

$$y'[0] = aa\, y[-h] + bb\, y[0] + cc\, y[h]$$

$$y'[0] = b = aa\, y[-h] + bb\, y[0] + cc\, y[h]$$
$$= aa(a - bh + ch^2) + bb\, a + cc(a - bh + ch^2)$$
$$= a(aa + bb + cc) + b\, h\, (cc - aa) + ch^2\, (aa + cc)$$

$$b = a(aa + bb + cc) + b\, h\, (cc - aa) + ch^2\, (aa + cc)$$

$$\left.\begin{array}{l} (aa + bb + cc) = 0 \\ h\,(cc - aa) = 1 \\ h^2(cc + aa) = 0 \rightarrow aa = -cc \end{array}\right\} \quad cc = \frac{1}{2\,h} \quad aa = -\frac{1}{2\,h} \quad bb = 0$$

$$y'[0] = \frac{y[h] - y[-h]}{2h} \qquad y'_i = \frac{y_{i+1} - y_{i-1}}{2h}$$

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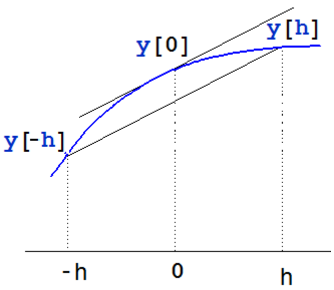
```
y[x_] := a + b x + c x^2
```



first derivative

```
dcy[x_] := aa y[-h] + bb y[0] + cc y[h]
dc = D[y[x], x] /. x → 0
```

```
dcy[0] - dc == 0
```

```
Solve[{Coefficient[dcy[x] - dc, a] == 0,
       Coefficient[dcy[x] - dc, b] == 0,
       Coefficient[dcy[x] - dc, c] == 0}, {aa, bb, cc}]
```

$$\left\{\left\{aa \to -\frac{1}{2\,h},\ bb \to 0,\ cc \to \frac{1}{2\,h}\right\}\right\}$$

second derivative

```
y[x_] := a + b x + c x^2
```

```
d2c = D[y[x], {x, 2}] /. x → 0;
d2cy[x_] := aa y[-h] + bb y[0] + cc y[h]
```

```
d2cy[0] - d2c == 0
```

```
Solve[{Coefficient[d2cy[x] - d2c, a] == 0,
       Coefficient[d2cy[x] - d2c, b] == 0,
       Coefficient[d2cy[x] - d2c, c] == 0}, {aa, bb, cc}]
```

$$\left\{\left\{aa \to \frac{1}{h^2},\ bb \to -\frac{2}{h^2},\ cc \to \frac{1}{h^2}\right\}\right\}$$

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```
y[x_] := a + b x + c x^2 + d x^3 + e x^4;

dc = D[y[x], x] /. x → 0;
d2c = D[y[x], {x, 2}] /. x → 0;

dcy[x_] := aa y[-2 h] + bb y[-h] + cc y[0] + dd y[h] + ee y[2 h];
d2cy[x_] := aa y[-2 h] + bb y[-h] + cc y[0] + dd y[h] + ee y[2 h];
```

```
Solve[{Coefficient[dcy[x] - dc, a] == 0,
      Coefficient[dcy[x] - dc, b] == 0,
      Coefficient[dcy[x] - dc, c] == 0,
      Coefficient[dcy[x] - dc, d] == 0,
      Coefficient[dcy[x] - dc, e] == 0}, {aa, bb, cc, dd, ee}]
```

$$\left\{\left\{aa \to \frac{1}{12\,h},\ bb \to -\frac{2}{3\,h},\ cc \to 0,\ dd \to \frac{2}{3\,h},\ ee \to -\frac{1}{12\,h}\right\}\right\}$$

```
Solve[{Coefficient[d2cy[x] - d2c, a] == 0,
      Coefficient[d2cy[x] - d2c, b] == 0,
      Coefficient[d2cy[x] - d2c, c] == 0,
      Coefficient[d2cy[x] - d2c, d] == 0,
      Coefficient[d2cy[x] - d2c, e] == 0}, {aa, bb, cc, dd, ee}]
```

$$\left\{\left\{aa \to -\frac{1}{12\,h^2},\ bb \to \frac{4}{3\,h^2},\ cc \to -\frac{5}{2\,h^2},\ dd \to \frac{4}{3\,h^2},\ ee \to -\frac{1}{12\,h^2}\right\}\right\}$$

BOUNDARY CONDITIONS