APPENDIX III

TABLE 57

DIRECT PRODUCTS OF REPRESENTATIONS (SPECIES) FOR ALL IMPORTANT POINT GROUPS

Species to be omitted in the symmetrical product of a degenerate species with itself are put in square brackets. They represent the antisymmetric product.

C_s

	A'	A''	$E_{\frac{1}{2}}$
A'	A'	A''	$E_{\frac{1}{2}}$
A''		A'	$E_{\frac{1}{2}}$
$E_{\frac{1}{2}}$			$[A'], A', A'', A''$

C_i, (C_1) [a]

	A_g	A_u	$B_{\frac{1}{2}g}$	$B_{\frac{1}{2}u}$
A_g	A_g	A_u	$B_{\frac{1}{2}g}$	$B_{\frac{1}{2}u}$
A_u		A_g	$B_{\frac{1}{2}u}$	$B_{\frac{1}{2}g}$
$B_{\frac{1}{2}g}$			A_g	A_u
$B_{\frac{1}{2}u}$				A_g

C_{2v},(C_2) [b],(C_{2h}) [b,c]

	A_1	A_2	B_1	B_2	$E_{\frac{1}{2}}$
A_1	A_1	A_2	B_1	B_2	$E_{\frac{1}{2}}$
A_2		A_1	B_2	B_1	$E_{\frac{1}{2}}$
B_1			A_1	A_2	$E_{\frac{1}{2}}$
B_2				A_1	$E_{\frac{1}{2}}$
$E_{\frac{1}{2}}$					$[A_1], A_2, B_1, B_2$

C_3

	A	E	$E_{\frac{1}{2}}$	$B_{\frac{3}{2}}$
A	A	E	$E_{\frac{1}{2}}$	$B_{\frac{3}{2}}$
E		$[A], A, E$	$E_{\frac{1}{2}}, 2B_{\frac{3}{2}}$	$E_{\frac{1}{2}}$
$E_{\frac{1}{2}}$			$[A], A, E$	E
$B_{\frac{3}{2}}$				A

D_2, (D_{2h}) [c]

	A	B_1	B_2	B_3	$E_{\frac{1}{2}}$
A	A	B_1	B_2	B_3	$E_{\frac{1}{2}}$
B_1		A	B_3	B_2	$E_{\frac{1}{2}}$
B_2			A	B_1	$E_{\frac{1}{2}}$
B_3				A	$E_{\frac{1}{2}}$
$E_{\frac{1}{2}}$					$[A], B_1, B_2, B_3$

D_3, C_{3v}, (D_{3d}) [c]

	A_1	A_2	E	$E_{\frac{1}{2}}$	$E_{\frac{3}{2}}$
A_1	A_1	A_2	E	$E_{\frac{1}{2}}$	$E_{\frac{3}{2}}$
A_2		A_1	E	$E_{\frac{1}{2}}$	$E_{\frac{3}{2}}$
E			$A_1, [A_2], E$	$E_{\frac{1}{2}}, E_{\frac{3}{2}}$	$E_{\frac{1}{2}}, E_{\frac{1}{2}}$
$E_{\frac{1}{2}}$				$[A_1], A_2, E$	E, E
$E_{\frac{3}{2}}$					$[A_1], A_2, A_1, A_2$

[a] For this point group g and u should be omitted. [b] For these point groups the subscripts 1 and 2 should be dropped.
[c] For these point groups the (g, u) rule must be added, that is $g \times g = g$, $g \times u = u$, $u \times u = g$.

TABLE 57 (*Continued*)

D_{3h}, (C_{3h})[b]

	A_1'	A_2'	A_1''	A_2''	E'	E''	$E_{\frac{1}{2}}$	$E_{\frac{3}{2}}$	$E_{\frac{5}{2}}$
A_1'	A_1'	A_2'	A_1''	A_2''	E'	E''	$E_{\frac{1}{2}}$	$E_{\frac{3}{2}}$	$E_{\frac{5}{2}}$
A_2'		A_1'	A_2''	A_1''	E'	E''	$E_{\frac{1}{2}}$	$E_{\frac{3}{2}}$	$E_{\frac{5}{2}}$
A_1''			A_1'	A_2'	E''	E'	$E_{\frac{5}{2}}$	$E_{\frac{3}{2}}$	$E_{\frac{1}{2}}$
A_2''				A_1'	E''	E'	$E_{\frac{5}{2}}$	$E_{\frac{3}{2}}$	$E_{\frac{1}{2}}$
E'					$A_1', [A_2'], E'$	A_1'', A_2'', E''	$E_{\frac{3}{2}}, E_{\frac{5}{2}}$	$E_{\frac{1}{2}}, E_{\frac{5}{2}}$	$E_{\frac{1}{2}}, E_{\frac{3}{2}}$
E''						$A_1', [A_2'], E'$	$E_{\frac{1}{2}}, E_{\frac{3}{2}}$	$E_{\frac{1}{2}}, E_{\frac{5}{2}}$	$E_{\frac{3}{2}}, E_{\frac{5}{2}}$
$E_{\frac{1}{2}}$							$[A_1'], A_2', E''$	E', E''	A_1'', A_2'', E'
$E_{\frac{3}{2}}$								$[A_1'], A_2', A_1'', A_2''$	E', E''
$E_{\frac{5}{2}}$									$[A_1'], A_2', E''$

D_4, C_{4v}, D_{2d}, (D_{4h})[c]

	A_1	A_2	B_1	B_2	E	$E_{\frac{1}{2}}$	$E_{\frac{3}{2}}$
A_1	A_1	A_2	B_1	B_2	E	$E_{\frac{1}{2}}$	$E_{\frac{3}{2}}$
A_2		A_1	B_2	B_1	E	$E_{\frac{1}{2}}$	$E_{\frac{3}{2}}$
B_1			A_1	A_2	E	$E_{\frac{3}{2}}$	$E_{\frac{1}{2}}$
B_2				A_1	E	$E_{\frac{3}{2}}$	$E_{\frac{1}{2}}$
E					$A_1, [A_2], B_1, B_2$	$E_{\frac{1}{2}}, E_{\frac{3}{2}}$	$E_{\frac{1}{2}}, E_{\frac{3}{2}}$
$E_{\frac{1}{2}}$						$[A_1], A_2, E$	B_1, B_2, E
$E_{\frac{3}{2}}$							$[A_1], A_2, E$

D_5, C_{5v}, (D_{5d})[c]

	A_1	A_2	E_1	E_2	$E_{\frac{1}{2}}$	$E_{\frac{3}{2}}$	$E_{\frac{5}{2}}$
A_1	A_1	A_2	E_1	E_2	$E_{\frac{1}{2}}$	$E_{\frac{3}{2}}$	$E_{\frac{5}{2}}$
A_2		A_1	E_1	E_2	$E_{\frac{1}{2}}$	$E_{\frac{3}{2}}$	$E_{\frac{5}{2}}$
E_1			$A_1, [A_2], E_2$	E_1, E_2	$E_{\frac{1}{2}}, E_{\frac{5}{2}}$	$E_{\frac{1}{2}}, E_{\frac{5}{2}}$	$E_{\frac{3}{2}}, E_{\frac{5}{2}}$
E_2				$A_1, [A_2], E_1$	$E_{\frac{3}{2}}, E_{\frac{5}{2}}$	$E_{\frac{1}{2}}, E_{\frac{3}{2}}$	$E_{\frac{3}{2}}, E_{\frac{5}{2}}$
$E_{\frac{1}{2}}$					$[A_1], A_2, E_1$	E_1, E_2	E_2, E_2
$E_{\frac{3}{2}}$						$[A_1], A_2, E_2$	E_1, E_1
$E_{\frac{5}{2}}$							$[A_1], A_2, A_1, A_2$

D_6, C_{6v}, (D_{6h})[c]

	A_1	A_2	B_1	B_2	E_1	E_2	$E_{\frac{1}{2}}$	$E_{\frac{3}{2}}$	$E_{\frac{5}{2}}$
A_1	A_1	A_2	B_1	B_2	E_1	E_2	$E_{\frac{1}{2}}$	$E_{\frac{3}{2}}$	$E_{\frac{5}{2}}$
A_2		A_1	B_2	B_1	E_1	E_2	$E_{\frac{1}{2}}$	$E_{\frac{3}{2}}$	$E_{\frac{5}{2}}$
B_1			A_1	A_2	E_2	E_1	$E_{\frac{5}{2}}$	$E_{\frac{3}{2}}$	$E_{\frac{1}{2}}$
B_2				A_1	E_2	E_1	$E_{\frac{5}{2}}$	$E_{\frac{3}{2}}$	$E_{\frac{1}{2}}$
E_1					$A_1, [A_2], E_2$	B_1, B_2, E_1	$E_{\frac{1}{2}}, E_{\frac{3}{2}}$	$E_{\frac{1}{2}}, E_{\frac{5}{2}}$	$E_{\frac{5}{2}}, E_{\frac{5}{2}}$
E_2						$A_1, [A_2], E_2$	$E_{\frac{3}{2}}, E_{\frac{5}{2}}$	$E_{\frac{1}{2}}, E_{\frac{5}{2}}$	$E_{\frac{1}{2}}, E_{\frac{3}{2}}$
$E_{\frac{1}{2}}$							$[A_1], A_2, E_1$	E_1, E_2	B_1, B_2, E_2
$E_{\frac{3}{2}}$								$[A_1], A_2, B_1, B_2$	E_1, E_2
$E_{\frac{5}{2}}$									$[A_1], A_2, E_1$

Table 57 (*Continued*)

D_8, C_{8v}, D_{4d}, (D_{8h})[b]

	A_1	A_2	B_1	B_2	E_1	E_2	E_3	$E_{\frac{1}{2}}$	$E_{\frac{3}{2}}$	$E_{\frac{5}{2}}$	$E_{\frac{7}{2}}$
A_1	A_1	A_2	B_1	B_2	E_1	E_2	E_3	$E_{\frac{1}{2}}$	$E_{\frac{3}{2}}$	$E_{\frac{5}{2}}$	$E_{\frac{7}{2}}$
A_2		A_1	B_2	B_1	E_1	E_2	E_3	$E_{\frac{1}{2}}$	$E_{\frac{3}{2}}$	$E_{\frac{5}{2}}$	$E_{\frac{7}{2}}$
B_1			A_1	A_2	E_3	E_2	E_1	$E_{\frac{7}{2}}$	$E_{\frac{5}{2}}$	$E_{\frac{3}{2}}$	$E_{\frac{1}{2}}$
B_2				A_1	E_3	E_2	E_1	$E_{\frac{7}{2}}$	$E_{\frac{5}{2}}$	$E_{\frac{3}{2}}$	$E_{\frac{1}{2}}$
E_1					$A_1, [A_2], E_2$	E_1, E_3	B_1, B_2, E_2	$E_{\frac{1}{2}}, E_{\frac{3}{2}}$	$E_{\frac{1}{2}}, E_{\frac{5}{2}}$	$E_{\frac{3}{2}}, E_{\frac{7}{2}}$	$E_{\frac{5}{2}}, E_{\frac{7}{2}}$
E_2						$A_1, [A_2], B_1, B_2$	E_1, E_3	$E_{\frac{3}{2}}, E_{\frac{5}{2}}$	$E_{\frac{1}{2}}, E_{\frac{7}{2}}$	$E_{\frac{1}{2}}, E_{\frac{7}{2}}$	$E_{\frac{3}{2}}, E_{\frac{5}{2}}$
E_3							$A_1, [A_2], E_2$	$E_{\frac{5}{2}}, E_{\frac{7}{2}}$	$E_{\frac{3}{2}}, E_{\frac{7}{2}}$	$E_{\frac{1}{2}}, E_{\frac{5}{2}}$	$E_{\frac{1}{2}}, E_{\frac{3}{2}}$
$E_{\frac{1}{2}}$								$[A_1], A_2, E_1$	E_1, E_2	E_2, E_3	B_1, B_2, E_3
$E_{\frac{3}{2}}$									$[A_1], A_2, E_3$	B_1, B_2, E_1	E_2, E_3
$E_{\frac{5}{2}}$										$[A_1], A_2, E_3$	E_1, E_2
$E_{\frac{7}{2}}$											$[A_1], A_2, E_1$

D_∞, $C_{\infty v}$, $(D_{\infty h})$[b]

	Σ^+	Σ^-	Π	Δ	Φ	...	$E_{\frac{1}{2}}$	$E_{\frac{3}{2}}$	$E_{\frac{5}{2}}$	
Σ^+	Σ^+	Σ^-	Π	Δ	Φ	...	$E_{\frac{1}{2}}$	$E_{\frac{3}{2}}$	$E_{\frac{5}{2}}$	...
Σ^-		Σ^+	Π	Δ	Φ	...	$E_{\frac{1}{2}}$	$E_{\frac{3}{2}}$	$E_{\frac{5}{2}}$	...
Π			$\Sigma^+, [\Sigma^-], \Delta$	Π, Φ	Δ, Γ	...	$E_{\frac{1}{2}}, E_{\frac{3}{2}}$	$E_{\frac{1}{2}}, E_{\frac{5}{2}}$	$E_{\frac{3}{2}}, E_{\frac{7}{2}}$	...
Δ				$\Sigma^+, [\Sigma^-], \Gamma$	Π, H	...	$E_{\frac{3}{2}}, E_{\frac{5}{2}}$	$E_{\frac{3}{2}}, E_{\frac{7}{2}}$	$E_{\frac{1}{2}}, E_{\frac{9}{2}}$	...
Φ					$\Sigma^+, [\Sigma^-], I$	...	$E_{\frac{5}{2}}, E_{\frac{7}{2}}$	$E_{\frac{3}{2}}, E_{\frac{9}{2}}$	$E_{\frac{1}{2}}, E_{\frac{11}{2}}$	...
. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .
$E_{\frac{1}{2}}$							$[\Sigma^+], \Sigma^-, \Pi$	Π, Δ	Δ, Φ	...
$E_{\frac{3}{2}}$								$[\Sigma^+], \Sigma^-, \Phi$	Π, Γ	...
$E_{\frac{5}{2}}$									$[\Sigma^+], \Sigma^-, H$	...
. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .

K_h : $[D_j \times D_j]^- = D_{2j-1} \oplus D_{2j-3} \oplus \cdots D_4$ ó D_0

Table 57 (*Continued*)

O, T_d, (O_h)[b]

	A_1	A_2	E	F_1	F_2	$E_{\frac{1}{2}}$	$E_{\frac{5}{2}}$	$G_{\frac{3}{2}}$
A_1	A_1	A_2	E	F_1	F_2	$E_{\frac{1}{2}}$	$E_{\frac{5}{2}}$	$G_{\frac{3}{2}}$
A_2		A_1	E	F_2	F_1	$E_{\frac{5}{2}}$	$E_{\frac{1}{2}}$	$G_{\frac{3}{2}}$
E			$A_1, [A_2], E$	F_1, F_2	E_1, F_2	$G_{\frac{3}{2}}$	$G_{\frac{3}{2}}$	$E_{\frac{1}{2}}, E_{\frac{5}{2}}, G_{\frac{3}{2}}$
F_1				$A_1, E, [F_1], F_2$	A_2, E, F_1, F_2	$E_{\frac{1}{2}}, G_{\frac{3}{2}}$	$E_{\frac{5}{2}}, G_{\frac{3}{2}}$	$E_{\frac{1}{2}}, E_{\frac{5}{2}}, 2G_{\frac{3}{2}}$
F_2					$A_1, E, [F_1], F_2$	$E_{\frac{5}{2}}, G_{\frac{3}{2}}$	$E_{\frac{1}{2}}, G_{\frac{3}{2}}$	$E_{\frac{1}{2}}, E_{\frac{5}{2}}, 2G_{\frac{3}{2}}$
$E_{\frac{1}{2}}$						$[A_1], F_1$	A_2, F_2	E, F_1, F_2
$E_{\frac{5}{2}}$							$[A_1], F_1$	E, F_1, F_2
$G_{\frac{3}{2}}$								$[A_1], A_2, [E], 2F_1, [F_2], F_2$

I, (I_h)[b]

	A	F_1	F_2	G	H	$E_{\frac{1}{2}}$	$E_{\frac{7}{2}}$	$G_{\frac{3}{2}}$	$I_{\frac{5}{2}}$
A	A	F_1	F_2	G	H	$E_{\frac{1}{2}}$	$E_{\frac{7}{2}}$	$G_{\frac{3}{2}}$	$I_{\frac{5}{2}}$
F_1		$A, [F_1], H$	G, H	F_2, G, H	F_1, F_2, G, H	$E_{\frac{1}{2}}, G_{\frac{3}{2}}$	$I_{\frac{5}{2}}$	$E_{\frac{1}{2}}, G_{\frac{3}{2}}, I_{\frac{5}{2}}$	$E_{\frac{7}{2}}, G_{\frac{3}{2}}, 2I_{\frac{5}{2}}$
F_2			$A, [F_2], H$	F_1, G, H	F_1, F_2, G, H	$I_{\frac{5}{2}}$	$E_{\frac{7}{2}}, G_{\frac{3}{2}}$	$E_{\frac{7}{2}}, G_{\frac{3}{2}}, I_{\frac{5}{2}}$	$E_{\frac{1}{2}}, G_{\frac{3}{2}}, 2I_{\frac{5}{2}}$
G				$A, [F_1, F_2], G, H$	$F_1, F_2, G, 2H$	$E_{\frac{7}{2}}, I_{\frac{5}{2}}$	$E_{\frac{1}{2}}, I_{\frac{5}{2}}$	$G_{\frac{3}{2}}, 2I_{\frac{5}{2}}$	$E_{\frac{1}{2}}, E_{\frac{7}{2}}, 2G_{\frac{3}{2}}, 2I_{\frac{5}{2}}$
H					$A, [F_1, F_2, G], G, 2H$	$G_{\frac{3}{2}}, I_{\frac{5}{2}}$	$G_{\frac{3}{2}}, I_{\frac{5}{2}}$	$E_{\frac{1}{2}}, E_{\frac{7}{2}}, G_{\frac{3}{2}}, 2I_{\frac{5}{2}}$	$E_{\frac{1}{2}}, E_{\frac{7}{2}}, 2G_{\frac{3}{2}}, 3I_{\frac{5}{2}}$
$E_{\frac{1}{2}}$						$[A], F_1$	G	F_1, H	F_2, G, H
$E_{\frac{7}{2}}$							$[A], F_2$	F_2, H	F_1, G, H
$G_{\frac{3}{2}}$								$[A], F_1, F_2, G, [H]$	$F_1, F_2, 2G, 2H$
$I_{\frac{5}{2}}$									$[A], 2F_1, 2F_2, [G], G, [2H], H$