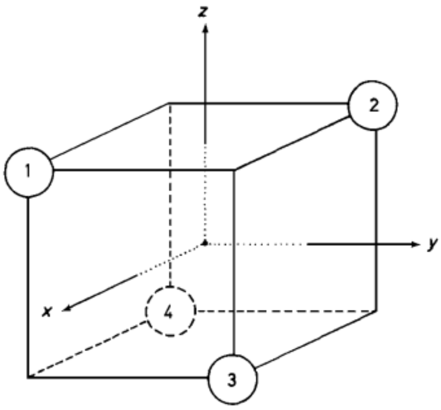

Els tres eixos han de coincidir amb els tres eixos binaris. Les coordenades dels vèrtexs són:

```
x1 = 1; x2 = -1; x3 = 1; x4 = -1;
y1 = -1;  y2 = 1; y3 = 1; y4 = -1;
z1 = 1; z2 = 1; z3 = -1; z4 = -1;
```

En termes del vèrtexs en cartesianes el volum del tetraedre ve donat per

$$V = \int_{\Omega^e} d\Omega^e = \frac{1}{6} \det \begin{bmatrix} 1 & 1 & 1 & 1 \\ x_1 & x_2 & x_3 & x_4 \\ y_1 & y_2 & y_3 & y_4 \\ z_1 & z_2 & z_3 & z_4 \end{bmatrix}$$

En el nostre cas:

```
m = {{1, 1, 1, 1}, {x1, x2, x3, x4}, {y1, y2, y3, y4}, {z1, z2, z3, z4}};
Det[m] / 6
```

$\frac{8}{3}$

La relació entre les coordenades cartesianes i les coordenades tetraèdriques és:

$$\begin{bmatrix} 1 \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ x_1 & x_2 & x_3 & x_4 \\ y_1 & y_2 & y_3 & y_4 \\ z_1 & z_2 & z_3 & z_4 \end{bmatrix} \begin{bmatrix} \zeta_1 \\ \zeta_2 \\ \zeta_3 \\ \zeta_4 \end{bmatrix}$$

En el nostre cas:

```
m.{t1, t2, t3, t4}

{t1 + t2 + t3 + t4, t1 - t2 + t3 - t4, -t1 + t2 + t3 - t4, t1 + t2 - t3 - t4}

x = t1 - t2 + t3 - t4; y = -t1 + t2 + t3 - t4; z = t1 + t2 - t3 - t4;
x * y * z // Expand
```

$-t1^3 + t1^2\, t2 + t1\, t2^2 - t2^3 + t1^2\, t3 - 2\, t1\, t2\, t3 + t2^2\, t3 + t1\, t3^2 + t2\, t3^2 - t3^3 + t1^2\, t4 - 2\, t1\, t2\, t4 + t2^2\, t4 - 2\, t1\, t3\, t4 - 2\, t2\, t3\, t4 + t3^2\, t4 + t1\, t4^2 + t2\, t4^2 + t3\, t4^2 - t4^3$

La integració analítica exacta sobre in tetraedre ve donada per una fórmula simple en terme de polinomis expressats en coordenades tetraèdriques:

$$\int_{\Omega^e} \zeta_1^i\, \zeta_2^j\, \zeta_3^k\, \zeta_4^\ell \; d\Omega^e = \frac{i!\; j!\; k!\; \ell!}{(i+j+k+\ell+3)!}\, 6V$$

En el nostre cas:

In[114]:= $\left(-\frac{3!}{(3+3)!}+\frac{2}{(1+2+3)!}+\frac{2}{(1+2+3)!}-\frac{3!}{(3+3)!}+\frac{2}{(1+2+3)!}-2\frac{1}{(1+1+1+3)!}+\frac{2}{(1+2+3)!}+\frac{2}{(1+2+3)!}+\frac{2}{(1+2+3)!}-\frac{3!}{(3+3)!}+\frac{2}{(1+2+3)!}-2\frac{1}{(1+1+1+3)!}+\frac{2}{(1+2+3)!}-2\frac{1}{(1+1+1+3)!}-2\frac{1}{(1+1+1+3)!}+\frac{2}{(1+2+3)!}+\frac{2}{(1+2+3)!}+\frac{2}{(1+2+3)!}+\frac{2}{(1+2+3)!}-\frac{3!}{(3+3)!}\right)6\,\mathrm{vol}$

Out[114]= $-\frac{8}{45}$

Atenció: és molt important definir be els eixos! Si fem que l'eix x vaja de 3 a 4 i l'eix y de l'1 al 2, la integració dóna zero:

In[19]:= $x1=-1;\ x2=0;\ x3=1;\ x4=0;$
$y1=0;\ y2=1;\ y3=0;\ y4=-1;$
$z1=-\frac{\sqrt{2}}{3};\ z2=2\frac{\sqrt{2}}{3};\ z3=-\frac{\sqrt{2}}{3};\ z4=2\frac{\sqrt{2}}{3};$
m = {{1, 1, 1, 1}, {x1, x2, x3, x4}, {y1, y2, y3, y4}, {z1, z2, z3, z4}};
Det[m] / 6

Out[23]= $-\frac{2\sqrt{2}}{3}$

In[24]:= m.{t1, t2, t3, t4}

Out[24]= $\left\{t1+t2+t3+t4,\ -t1+t3,\ t2-t4,\ -\frac{\sqrt{2}\ t1}{3}+\frac{2\sqrt{2}\ t2}{3}-\frac{\sqrt{2}\ t3}{3}+\frac{2\sqrt{2}\ t4}{3}\right\}$

In[25]:= $x=-t1+t3;\ y=t2-t4;\ z=-\frac{\sqrt{2}\ t1}{3}+\frac{2\sqrt{2}\ t2}{3}-\frac{\sqrt{2}\ t3}{3}+\frac{2\sqrt{2}\ t4}{3};$
x * y * z // Expand

Out[26]= $\frac{1}{3}\sqrt{2}\ t1^2\ t2-\frac{2}{3}\sqrt{2}\ t1\ t2^2+\frac{2}{3}\sqrt{2}\ t2^2\ t3-\frac{1}{3}\sqrt{2}\ t2\ t3^2-\frac{1}{3}\sqrt{2}\ t1^2\ t4+\frac{1}{3}\sqrt{2}\ t3^2\ t4+\frac{2}{3}\sqrt{2}\ t1\ t4^2-\frac{2}{3}\sqrt{2}\ t3\ t4^2$

In[27]:= $\frac{1}{3}\sqrt{2}\frac{2}{(1+2+3)!}-\frac{2}{3}\sqrt{2}\frac{2}{(1+2+3)!}+\frac{2}{3}\sqrt{2}\frac{2}{(1+2+3)!}-\frac{1}{3}\sqrt{2}\frac{2}{(1+2+3)!}-\frac{1}{3}\sqrt{2}\frac{2}{(1+2+3)!}+\frac{1}{3}\sqrt{2}\frac{2}{(1+2+3)!}+\frac{2}{3}\sqrt{2}\frac{2}{(1+2+3)!}-\frac{2}{3}\sqrt{2}\frac{2}{(1+2+3)!}$

Out[27]= 0