Càlcul de les fórmules de la derivada en diferències finites

```
(* Quoficients per a la derivada central a cinc punts *)

(* Hipòtesi: la fórmula ha de ser exacta per al polinomi
   y=a + b x + c x^2 +d x^3 + e x^4        *)

(* Hipòtesi: y"(i)= aa y(i+2) + bb y(i+1) + cc y(i) +dd y(i-1) + ee y(i-2)     *)


(*      y(i+2) = a+b(2 h)+c(2h)^2+d(2h)^3 +e(2h)^4
    y(i+1) = a+b h+c h^2+d h^3 +e h^4
    y(i)=a
     y(i-1) = a-b h+c h^2-d h^3 +e h^4
    y(i+2) = a-b(2 h)+c(2h)^2-d(2h)^3 +e(2h)^4
    y"(i)=2 c
 EN CONSEQUÈNCIA:

 2 c = aa (a+ 2b h+4 c h^2+ 8d h^3 +16 e h^4 )
     + bb (a+b h+c h^2+d h^3 +e h^4) + cc (a)
     + dd (a-b h+c h^2-d h^3 +e h^4)
     + ee (a - 2b h + 4 c h^2 - 8d h^3 + 16 e h^4)
       *)
Solve[{(aa + bb + cc + dd + ee) == 0,
       (2 aa + bb - dd - 2 ee) == 0,
       h^2 (4 aa + bb + dd + 4 ee) == 2 ,
      (8 aa + bb - dd - 8 ee) == 0,
      (16 aa + bb + dd + 16 ee) == 0},   {aa, bb, cc, dd, ee}]
```

$$\left\{\left\{cc \to -\frac{5}{2\,h^2},\ aa \to -\frac{1}{12\,h^2},\ bb \to \frac{4}{3\,h^2},\ dd \to \frac{4}{3\,h^2},\ ee \to -\frac{1}{12\,h^2}\right\}\right\}$$

```
aa = ee → -1 / 12
bb = dd →  4 / 3 → 16 / 12
cc → 5 / 2 →  -30 / 12
```

```
(* segona derivada central: de manera més compacta  *)

ClearAll["Global`*"]

y[x_] := a + b x + c x^2 + d x^3 + e x^4;

d2cy[x_] := aa y[-2 h] + bb y[-h] + cc y[0] + dd y[h] + ee y[2 h]

d2c = D[y[x], {x, 2}] /. x → 0;

Solve[{Coefficient[d2cy[x] - d2c, a] == 0, Coefficient[d2cy[x] - d2c, b] == 0,
  Coefficient[d2cy[x] - d2c, c] == 0, Coefficient[d2cy[x] - d2c, d] == 0,
  Coefficient[d2cy[x] - d2c, e] == 0}, {aa, bb, cc, dd, ee}]
```

$$\left\{\left\{cc \to -\frac{5}{2\,h^2},\ aa \to -\frac{1}{12\,h^2},\ bb \to \frac{4}{3\,h^2},\ dd \to \frac{4}{3\,h^2},\ ee \to -\frac{1}{12\,h^2}\right\}\right\}$$

Fórmules de la primera derivada en diferències finites

```
(* primera derivada central amb 5 punts  *)

ClearAll["Global`*"]

y[x_] := a + b x + c x^2 + d x^3 + e x^4;

dcy[x_] := aa y[-2 h] + bb y[-h] + cc y[0] + dd y[h] + ee y[2 h]

dc = D[y[x], {x, 1}] /. x → 0;

Solve[{Coefficient[dcy[x] - dc, a] == 0, Coefficient[dcy[x] - dc, b] == 0,
  Coefficient[dcy[x] - dc, c] == 0, Coefficient[dcy[x] - dc, d] == 0,
  Coefficient[dcy[x] - dc, e] == 0}, {aa, bb, cc, dd, ee}]
```

$$\left\{\left\{cc \to 0,\ aa \to \frac{1}{12\,h},\ bb \to -\frac{2}{3\,h},\ dd \to \frac{2}{3\,h},\ ee \to -\frac{1}{12\,h}\right\}\right\}$$

```
(* primera derivada central a set punts *)

(* Hipòtesi: la fórmula ha de ser exacta per al polinomi de grau 6 *)
(* Hipòtesi: y'(i)= aa y(i+3) + bb y(i+2) +
  cc y(i+1) + dd y(i) + ee y(i-1)  + ff y(i-2) + gg y(i-3)   *)

y[x_] = a + b x + c x^2 + d x^3 + e x^4 + f x^5 + g x^6 ;
dc = D[y[x], {x, 1}] /. x → 0;
dcy[x_] := aa y[-3 h] + bb y[-2 h] + cc y[-h] + dd y[0] + ee y[h] + ff y[2 h] + gg y[3 h];
Solve[{Coefficient[dcy[x] - dc, a] == 0, Coefficient[dcy[x] - dc, b] == 0,
  Coefficient[dcy[x] - dc, c] == 0, Coefficient[dcy[x] - dc, d] == 0,
  Coefficient[dcy[x] - dc, e] == 0, Coefficient[dcy[x] - dc, f] == 0,
  Coefficient[dcy[x] - dc, g] == 0}, {aa, bb, cc, dd, ee, ff, gg}]
```

$$\left\{\left\{dd \to 0,\ aa \to -\frac{1}{60\,h},\ bb \to \frac{3}{20\,h},\ cc \to -\frac{3}{4\,h},\ ee \to \frac{3}{4\,h},\ ff \to -\frac{3}{20\,h},\ gg \to \frac{1}{60\,h}\right\}\right\}$$

```
(* primera derivada central a 15 punts *)

y[x_] = a + b x + c x^2 + d x^3 + e x^4 + f x^5 +
    g x^6 + m x^7 + n x^8 + p x^9 + q x^10 + r x^11 + s x^12 + t x^13 + u x^14;
dc = D[y[x], {x, 1}] /. x → 0;
dcy[x_] :=
  aa y[-7 h] + bb y[-6 h] + cc y[-5 h] + dd y[-4 h] + ee y[-3 h] + ff y[-2 h] + gg y[-h] +
   mm y[0] + nn y[h] + pp y[2 h] + qq y[3 h] + rr y[4 h] + ss y[5 h] + tt y[6 h] + uu y[7 h];
Solve[{Coefficient[dcy[x] - dc, a] == 0, Coefficient[dcy[x] - dc, b] == 0,
  Coefficient[dcy[x] - dc, c] == 0, Coefficient[dcy[x] - dc, d] == 0,
  Coefficient[dcy[x] - dc, e] == 0, Coefficient[dcy[x] - dc, f] == 0,
  Coefficient[dcy[x] - dc, g] == 0, Coefficient[dcy[x] - dc, m] == 0,
  Coefficient[dcy[x] - dc, n] == 0, Coefficient[dcy[x] - dc, p] == 0,
  Coefficient[dcy[x] - dc, q] == 0, Coefficient[dcy[x] - dc, r] == 0,
  Coefficient[dcy[x] - dc, s] == 0, Coefficient[dcy[x] - dc, t] == 0,
  Coefficient[dcy[x] - dc, u] == 0},
 {aa, bb, cc, dd, ee, ff, gg, mm, nn, pp, qq, rr, ss, tt, uu}]
```

$$\left\{\left\{mm \to 0,\ aa \to -\frac{1}{24\,024\,h},\ bb \to \frac{7}{10\,296\,h},\ cc \to -\frac{7}{1320\,h},\right.\right.$$
$$dd \to \frac{7}{264\,h},\ ee \to -\frac{7}{72\,h},\ ff \to \frac{7}{24\,h},\ gg \to -\frac{7}{8\,h},\ nn \to \frac{7}{8\,h},\ pp \to -\frac{7}{24\,h},$$
$$\left.\left.qq \to \frac{7}{72\,h},\ rr \to -\frac{7}{264\,h},\ ss \to \frac{7}{1320\,h},\ tt \to -\frac{7}{10\,296\,h},\ uu \to \frac{1}{24\,024\,h}\right\}\right\}$$

Fórmules de la segon derivada en diferències finites

```
(* Quoficients per a la segon derivada central a 3 punts *)

(* Hipòtesi: la fórmula ha de ser exacta per al polinomi de grau 3 *)
(* Hipòtesi: y"(i)= aa y(i+1) + bb y(i) + cc y(i-1)   *)

y[x_] = a + b x + c x^2 ;
dc = D[y[x], {x, 2}] /. x → 0;
dcy[x_] := aa y[-h] + bb y[0] + cc y[h];
Solve[{Coefficient[dcy[x] - dc, a] == 0,
  Coefficient[dcy[x] - dc, b] == 0, Coefficient[dcy[x] - dc, c] == 0}, {aa, bb, cc}]
```

$$\left\{\left\{bb \to -\frac{2}{h^2},\ aa \to \frac{1}{h^2},\ cc \to \frac{1}{h^2}\right\}\right\}$$

```
(* Quoficients per a la segon derivada central a 5 punts *)

(* Hipòtesi: la fórmula ha de ser exacta per al polinomi de grau 4 *)
(* Hipòtesi: y"(i)= aa y(i+2) + bb y(i+1) + cc y(i) + dd y(i-1) + ee y(i-2)   *)

y[x_] = a + b x + c x^2 + d x^3 + e x^4 ;
dc = D[y[x], {x, 2}] /. x → 0;
dcy[x_] := aa y[-2 h] + bb y[- h] + cc y[0] + dd y[h] + ee y[2 h] ;
Solve[{Coefficient[dcy[x] - dc, a] == 0, Coefficient[dcy[x] - dc, b] == 0,
  Coefficient[dcy[x] - dc, c] == 0, Coefficient[dcy[x] - dc, d] == 0,
  Coefficient[dcy[x] - dc, e] == 0}, {aa, bb, cc, dd, ee}]
```

$$\left\{\left\{cc \to -\frac{5}{2\,h^2},\ aa \to -\frac{1}{12\,h^2},\ bb \to \frac{4}{3\,h^2},\ dd \to \frac{4}{3\,h^2},\ ee \to -\frac{1}{12\,h^2}\right\}\right\}$$

```
(* Quoficients per a la segon derivada central a set punts *)

(* Hipòtesi: la fórmula ha de ser exacta per al polinomi de grau 6 *)
(* Hipòtesi: y"(i)= aa y(i+3) + bb y(i+2) + cc
    y(i+1) + dd y(i) + ee y(i-1)  + ff y(i-2) + gg y(i-3)   *)

y[x_] = a + b x + c x^2 + d x^3 + e x^4 + f x^5 + g x^6 ;
dc = D[y[x], {x, 2}] /. x → 0;
dcy[x_] := aa y[-3 h] + bb y[-2 h] + cc y[-h] + dd y[0] + ee y[h] + ff y[2 h] + gg y[3 h];
Solve[{Coefficient[dcy[x] - dc, a] == 0, Coefficient[dcy[x] - dc, b] == 0,
  Coefficient[dcy[x] - dc, c] == 0, Coefficient[dcy[x] - dc, d] == 0,
  Coefficient[dcy[x] - dc, e] == 0, Coefficient[dcy[x] - dc, f] == 0,
  Coefficient[dcy[x] - dc, g] == 0}, {aa, bb, cc, dd, ee, ff, gg}]
```

$$\left\{\left\{dd \to -\frac{49}{18\,h^2},\ aa \to \frac{1}{90\,h^2},\ bb \to -\frac{3}{20\,h^2},\ cc \to \frac{3}{2\,h^2},\ ee \to \frac{3}{2\,h^2},\ ff \to -\frac{3}{20\,h^2},\ gg \to \frac{1}{90\,h^2}\right\}\right\}$$

```
(* Quoficients per a la segon derivada central a nou punts *)

(* Hipòtesi: la fórmula ha de ser exacta per al polinomi de grau 6 *)
(* Hipòtesi:
 y"(i)= aa y(i+4) + bb y(i+3) + cc y(i+2) + dd y(i+1) + ee y(i)  + ff y(i-1)
    + gg y(i-2) + mm y(i-3) + nn y(i-4)    *)

y[x_] = a + b x + c x^2 + d x^3 + e x^4 + f x^5 + g x^6 + m x^7 + n x^8;
dc = D[y[x], {x, 2}] /. x → 0;
dcy[x_] := aa y[-4 h] + bb y[-3 h] + cc y[-2 h] +
   dd y[-h] + ee y[0] + ff y[h] + gg y[2 h] + mm y[3 h] + nn y[4 h];
Solve[{Coefficient[dcy[x] - dc, a] == 0, Coefficient[dcy[x] - dc, b] == 0,
  Coefficient[dcy[x] - dc, c] == 0, Coefficient[dcy[x] - dc, d] == 0,
  Coefficient[dcy[x] - dc, e] == 0, Coefficient[dcy[x] - dc, f] == 0,
  Coefficient[dcy[x] - dc, g] == 0, Coefficient[dcy[x] - dc, m] == 0,
  Coefficient[dcy[x] - dc, n] == 0}, {aa, bb, cc, dd, ee, ff, gg, mm, nn}]
```

$$\left\{\left\{ee \to -\frac{205}{72\,h^2},\ aa \to -\frac{1}{560\,h^2},\ bb \to \frac{8}{315\,h^2},\ cc \to -\frac{1}{5\,h^2},\right.\right.$$
$$\left.\left.dd \to \frac{8}{5\,h^2},\ ff \to \frac{8}{5\,h^2},\ gg \to -\frac{1}{5\,h^2},\ mm \to \frac{8}{315\,h^2},\ nn \to -\frac{1}{560\,h^2}\right\}\right\}$$

```
(* Quoficients per a la segona derivada central a 11 punts *)

(* Hipòtesi:
 y"(i)= aa y(i+5) + bb y(i+4) + cc y(i+3) + dd y(i+2) + ee y(i+1)  + ff y(i)
    + gg y(i-1) + mm y(i-2) + nn y(i-3)
                    + pp y(i-4)+ qq y(i-5)    *)

y[x_] = a + b x + c x^2 + d x^3 + e x^4 + f x^5 + g x^6 + m x^7 + n x^8 + p x^9 + q x^10;
dc = D[y[x], {x, 2}] /. x → 0;
dcy[x_] := aa y[-5 h] + bb y[-4 h] + cc y[-3 h] + dd y[-2 h] +
   ee y[-h] + ff y[0] + gg y[h] + mm y[2 h] + nn y[3 h] + pp y[4 h] + qq y[5 h];
Solve[{Coefficient[dcy[x] - dc, a] == 0, Coefficient[dcy[x] - dc, b] == 0,
  Coefficient[dcy[x] - dc, c] == 0, Coefficient[dcy[x] - dc, d] == 0,
  Coefficient[dcy[x] - dc, e] == 0, Coefficient[dcy[x] - dc, f] == 0,
  Coefficient[dcy[x] - dc, g] == 0, Coefficient[dcy[x] - dc, m] == 0,
  Coefficient[dcy[x] - dc, n] == 0, Coefficient[dcy[x] - dc, p] == 0,
  Coefficient[dcy[x] - dc, q] == 0}, {aa, bb, cc, dd, ee, ff, gg, mm, nn, pp, qq}]
```

$$\left\{\left\{ff \to -\frac{5269}{1800\,h^2},\ aa \to \frac{1}{3150\,h^2},\ bb \to -\frac{5}{1008\,h^2},\ cc \to \frac{5}{126\,h^2},\ dd \to -\frac{5}{21\,h^2},\right.\right.$$
$$\left.\left.ee \to \frac{5}{3\,h^2},\ gg \to \frac{5}{3\,h^2},\ mm \to -\frac{5}{21\,h^2},\ nn \to \frac{5}{126\,h^2},\ pp \to -\frac{5}{1008\,h^2},\ qq \to \frac{1}{3150\,h^2}\right\}\right\}$$

```
(* Quoficients per a la segona derivada central a 13 punts *)

y[x_] = a + b x + c x^2 + d x^3 + e x^4 + f x^5 + g x^6 + m x^7 + n x^8 + p x^9 + q x^10 + r x^11 + s x^12;
dc = D[y[x], {x, 2}] /. x → 0;
dcy[x_] := aa y[-6 h] + bb y[-5 h] + cc y[-4 h] + dd y[-3 h] + ee y[-2 h] + ff y[-h] +
    gg y[0] + mm y[h] + nn y[2 h] + pp y[3 h] + qq y[4 h] + rr y[5 h] + ss y[6 h];
Solve[{Coefficient[dcy[x] - dc, a] == 0, Coefficient[dcy[x] - dc, b] == 0,
  Coefficient[dcy[x] - dc, c] == 0, Coefficient[dcy[x] - dc, d] == 0,
  Coefficient[dcy[x] - dc, e] == 0, Coefficient[dcy[x] - dc, f] == 0,
  Coefficient[dcy[x] - dc, g] == 0, Coefficient[dcy[x] - dc, m] == 0,
  Coefficient[dcy[x] - dc, n] == 0, Coefficient[dcy[x] - dc, p] == 0,
  Coefficient[dcy[x] - dc, q] == 0, Coefficient[dcy[x] - dc, r] == 0,
  Coefficient[dcy[x] - dc, s] == 0}, {aa, bb, cc, dd, ee, ff, gg, mm, nn, pp, qq, rr, ss}]
```

$$\left\{\left\{gg \to -\frac{5369}{1800\,h^2},\ aa \to -\frac{1}{16\,632\,h^2},\ bb \to \frac{2}{1925\,h^2},\right.\right.$$
$$cc \to -\frac{1}{112\,h^2},\ dd \to \frac{10}{189\,h^2},\ ee \to -\frac{15}{56\,h^2},\ ff \to \frac{12}{7\,h^2},\ mm \to \frac{12}{7\,h^2},$$
$$\left.\left.nn \to -\frac{15}{56\,h^2},\ pp \to \frac{10}{189\,h^2},\ qq \to -\frac{1}{112\,h^2},\ rr \to \frac{2}{1925\,h^2},\ ss \to -\frac{1}{16\,632\,h^2}\right\}\right\}$$

```
(* Quoficients per a la segona derivada central a 15 punts *)

y[x_] = a + b x + c x^2 + d x^3 + e x^4 + f x^5 +
    g x^6 + m x^7 + n x^8 + p x^9 + q x^10 + r x^11 + s x^12 + t x^13 + u x^14;
dc = D[y[x], {x, 2}] /. x → 0;
dcy[x_] :=
  aa y[-7 h] + bb y[-6 h] + cc y[-5 h] + dd y[-4 h] + ee y[-3 h] + ff y[-2 h] + gg y[-h] +
   mm y[0] + nn y[h] + pp y[2 h] + qq y[3 h] + rr y[4 h] + ss y[5 h] + tt y[6 h] + uu y[7 h];
Solve[{Coefficient[dcy[x] - dc, a] == 0, Coefficient[dcy[x] - dc, b] == 0,
  Coefficient[dcy[x] - dc, c] == 0, Coefficient[dcy[x] - dc, d] == 0,
  Coefficient[dcy[x] - dc, e] == 0, Coefficient[dcy[x] - dc, f] == 0,
  Coefficient[dcy[x] - dc, g] == 0, Coefficient[dcy[x] - dc, m] == 0,
  Coefficient[dcy[x] - dc, n] == 0, Coefficient[dcy[x] - dc, p] == 0,
  Coefficient[dcy[x] - dc, q] == 0, Coefficient[dcy[x] - dc, r] == 0,
  Coefficient[dcy[x] - dc, s] == 0, Coefficient[dcy[x] - dc, t] == 0,
  Coefficient[dcy[x] - dc, u] == 0},
 {aa, bb, cc, dd, ee, ff, gg, mm, nn, pp, qq, rr, ss, tt, uu}]
```

$$\left\{\left\{mm \to -\frac{266\,681}{88\,200\,h^2},\ aa \to \frac{1}{84\,084\,h^2},\ bb \to -\frac{7}{30\,888\,h^2},\ cc \to \frac{7}{3300\,h^2},\right.\right.$$
$$dd \to -\frac{7}{528\,h^2},\ ee \to \frac{7}{108\,h^2},\ ff \to -\frac{7}{24\,h^2},\ gg \to \frac{7}{4\,h^2},\ nn \to \frac{7}{4\,h^2},\ pp \to -\frac{7}{24\,h^2},$$
$$\left.\left.qq \to \frac{7}{108\,h^2},\ rr \to -\frac{7}{528\,h^2},\ ss \to \frac{7}{3300\,h^2},\ tt \to -\frac{7}{30\,888\,h^2},\ uu \to \frac{1}{84\,084\,h^2}\right\}\right\}$$

```
(* Quoficients per a la segona derivada central a 17 punts *)

y[x_] = a + b x + c x^2 + d x^3 + e x^4 + f x^5 + g x^6 +
    m x^7 + n x^8 + p x^9 + q x^10 + r x^11 + s x^12 + t x^13 + u x^14 + v x^15 + z x^16;
dc = D[y[x], {x, 2}] /. x → 0;
dcy[x_] := aa y[-8 h] + bb y[-7 h] + cc y[-6 h] + dd y[-5 h] +
    ee y[-4 h] + ff y[-3 h] + gg y[-2 h] + mm y[-h] + nn y[0] + pp y[h] +
    qq y[2 h] + rr y[3 h] + ss y[4 h] + tt y[5 h] + uu y[6 h] + vv y[7 h] + zz y[8 h];
Solve[{Coefficient[dcy[x] - dc, a] == 0, Coefficient[dcy[x] - dc, b] == 0,
  Coefficient[dcy[x] - dc, c] == 0,
  Coefficient[dcy[x] - dc, d] == 0, Coefficient[dcy[x] - dc, e] == 0,
  Coefficient[dcy[x] - dc, f] == 0, Coefficient[dcy[x] - dc, g] == 0,
  Coefficient[dcy[x] - dc, m] == 0, Coefficient[dcy[x] - dc, n] == 0,
  Coefficient[dcy[x] - dc, p] == 0, Coefficient[dcy[x] - dc, q] == 0,
  Coefficient[dcy[x] - dc, r] == 0, Coefficient[dcy[x] - dc, s] == 0,
  Coefficient[dcy[x] - dc, t] == 0, Coefficient[dcy[x] - dc, u] == 0,
  Coefficient[dcy[x] - dc, v] == 0, Coefficient[dcy[x] - dc, z] == 0},
 {aa, bb, cc, dd, ee, ff, gg, mm, nn, pp, qq, rr, ss, tt, uu, vv, zz}]
```

$$\left\{\left\{nn \to -\frac{1\,077\,749}{352\,800\,h^2},\ aa \to -\frac{1}{411\,840\,h^2},\ bb \to \frac{16}{315\,315\,h^2},\ cc \to -\frac{2}{3861\,h^2},\ dd \to \frac{112}{32\,175\,h^2},\right.\right.$$
$$ee \to -\frac{7}{396\,h^2},\ ff \to \frac{112}{1485\,h^2},\ gg \to -\frac{14}{45\,h^2},\ mm \to \frac{16}{9\,h^2},\ pp \to \frac{16}{9\,h^2},\ qq \to -\frac{14}{45\,h^2},\ rr \to \frac{112}{1485\,h^2},$$
$$\left.\left.ss \to -\frac{7}{396\,h^2},\ tt \to \frac{112}{32\,175\,h^2},\ uu \to -\frac{2}{3861\,h^2},\ vv \to \frac{16}{315\,315\,h^2},\ zz \to -\frac{1}{411\,840\,h^2}\right\}\right\}$$

```
(* Quoficients per a la segona derivada central a 19 punts *)

y[x_] = a + b x + c x^2 + d x^3 + e x^4 + f x^5 + g x^6 + m x^7 + n x^8 +
    p x^9 + q x^10 + r x^11 + s x^12 + t x^13 + u x^14 + v x^15 + z x^16 + j x^17 + k x^18;
dc = D[y[x], {x, 2}] /. x → 0;
dcy[x_] := aa y[-9 h] + bb y[-8 h] + cc y[-7 h] + dd y[-6 h] + ee y[-5 h] +
    ff y[-4 h] + gg y[-3 h] + mm y[-2 h] + nn y[-h] + pp y[0] + qq y[h] + rr y[2 h] +
    ss y[3 h] + tt y[4 h] + uu y[5 h] + vv y[6 h] + zz y[7 h] + jj y[8 h] + kk y[9 h];
Solve[{Coefficient[dcy[x] - dc, a] == 0, Coefficient[dcy[x] - dc, b] == 0,
  Coefficient[dcy[x] - dc, c] == 0,
  Coefficient[dcy[x] - dc, d] == 0, Coefficient[dcy[x] - dc, e] == 0,
  Coefficient[dcy[x] - dc, f] == 0, Coefficient[dcy[x] - dc, g] == 0,
  Coefficient[dcy[x] - dc, m] == 0, Coefficient[dcy[x] - dc, n] == 0,
  Coefficient[dcy[x] - dc, p] == 0, Coefficient[dcy[x] - dc, q] == 0,
  Coefficient[dcy[x] - dc, r] == 0, Coefficient[dcy[x] - dc, s] == 0,
  Coefficient[dcy[x] - dc, t] == 0, Coefficient[dcy[x] - dc, u] == 0,
  Coefficient[dcy[x] - dc, v] == 0, Coefficient[dcy[x] - dc, z] == 0,
  Coefficient[dcy[x] - dc, j] == 0, Coefficient[dcy[x] - dc, k] == 0},
 {aa, bb, cc, dd, ee, ff, gg, mm, nn, pp, qq, rr, ss, tt, uu, vv, zz, jj, kk}]
```

$$\left\{\left\{pp \to -\frac{9\,778\,141}{3\,175\,200\,h^2},\ aa \to \frac{1}{1\,969\,110\,h^2},\ bb \to -\frac{9}{777\,920\,h^2},\ cc \to \frac{9}{70\,070\,h^2},\right.\right.$$
$$dd \to -\frac{2}{2145\,h^2},\ ee \to \frac{18}{3575\,h^2},\ ff \to -\frac{63}{2860\,h^2},\ gg \to \frac{14}{165\,h^2},\ mm \to -\frac{18}{55\,h^2},$$
$$nn \to \frac{9}{5\,h^2},\ qq \to \frac{9}{5\,h^2},\ rr \to -\frac{18}{55\,h^2},\ ss \to \frac{14}{165\,h^2},\ tt \to -\frac{63}{2860\,h^2},\ uu \to \frac{18}{3575\,h^2},$$
$$\left.\left.vv \to -\frac{2}{2145\,h^2},\ zz \to \frac{9}{70\,070\,h^2},\ jj \to -\frac{9}{777\,920\,h^2},\ kk \to \frac{1}{1\,969\,110\,h^2}\right\}\right\}$$

```
(* Quoficients per a la segona derivada central a 21 punts *)

y[x_] = a + b x + c x^2 + d x^3 + e x^4 + f x^5 + g x^6 + m x^7 + n x^8 + p x^9 + q x^10 +
    r x^11 + s x^12 + t x^13 + u x^14 + v x^15 + z x^16 + j x^17 + k x^18 + ab x^19 + ac x^20;
dc = D[y[x], {x, 2}] /. x → 0;
dcy[x_] := aa y[-10 h] + bb y[-9 h] + cc y[-8 h] + dd y[-7 h] + ee y[-6 h] + ff y[-5 h] +
    gg y[-4 h] + mm y[-3 h] + nn y[-2 h] + pp y[-h] + qq y[0] + rr y[h] + ss y[2 h] + tt y[3 h] +
    uu y[4 h] + vv y[5 h] + zz y[6 h] + jj y[7 h] + kk y[8 h] + abab y[9 h] + acac y[10 h];
Solve[{Coefficient[dcy[x] - dc, a] == 0, Coefficient[dcy[x] - dc, b] == 0,
  Coefficient[dcy[x] - dc, c] == 0, Coefficient[dcy[x] - dc, d] == 0,
  Coefficient[dcy[x] - dc, e] == 0, Coefficient[dcy[x] - dc, f] == 0,
  Coefficient[dcy[x] - dc, g] == 0, Coefficient[dcy[x] - dc, m] == 0,
  Coefficient[dcy[x] - dc, n] == 0, Coefficient[dcy[x] - dc, p] == 0,
  Coefficient[dcy[x] - dc, q] == 0, Coefficient[dcy[x] - dc, r] == 0,
  Coefficient[dcy[x] - dc, s] == 0, Coefficient[dcy[x] - dc, t] == 0,
  Coefficient[dcy[x] - dc, u] == 0, Coefficient[dcy[x] - dc, v] == 0,
  Coefficient[dcy[x] - dc, z] == 0, Coefficient[dcy[x] - dc, j] == 0,
  Coefficient[dcy[x] - dc, k] == 0, Coefficient[dcy[x] - dc, ab] == 0,
  Coefficient[dcy[x] - dc, ac] == 0},
 {aa, bb, cc, dd, ee, ff, gg, mm, nn, pp, qq, rr, ss, tt, uu, vv, zz, jj, kk, abab, acac}]
```

$$\left\{\left\{qq \to -\frac{1\,968\,329}{635\,040\,h^2},\ aa \to -\frac{1}{9\,237\,800\,h^2},\ bb \to \frac{10}{3\,741\,309\,h^2},\ cc \to -\frac{5}{155\,584\,h^2},\ dd \to \frac{30}{119\,119\,h^2},\right.\right.$$
$$ee \to -\frac{5}{3432\,h^2},\ ff \to \frac{24}{3575\,h^2},\ gg \to -\frac{15}{572\,h^2},\ mm \to \frac{40}{429\,h^2},\ nn \to -\frac{15}{44\,h^2},\ pp \to \frac{20}{11\,h^2},$$
$$rr \to \frac{20}{11\,h^2},\ ss \to -\frac{15}{44\,h^2},\ tt \to \frac{40}{429\,h^2},\ uu \to -\frac{15}{572\,h^2},\ vv \to \frac{24}{3575\,h^2},\ zz \to -\frac{5}{3432\,h^2},$$
$$\left.\left.jj \to \frac{30}{119\,119\,h^2},\ kk \to -\frac{5}{155\,584\,h^2},\ abab \to \frac{10}{3\,741\,309\,h^2},\ acac \to -\frac{1}{9\,237\,800\,h^2}\right\}\right\}$$

```
(*-------------------------------------------------------------------------------
*)
```

Fórmules de la tercer derivada en diferències finites

```
(* Quoficients per a la tercera derivada central a 3 punts (aquesta NO existeix!) *)

(* Hipòtesi: la fórmula ha de ser exacta per al polinomi de grau 3 *)
(* Hipòtesi: y'''(i)= aa y(i+1) + bb y(i) + cc y(i-1)    *)

y[x_] = a + b x + c x^2 ;
dc = D[y[x], {x, 3}] /. x → 0;
dcy[x_] := aa y[-2 h] + bb y[- h] + cc y[0] ;
Solve[{Coefficient[dcy[x] - dc, a] == 0,
  Coefficient[dcy[x] - dc, b] == 0, Coefficient[dcy[x] - dc, c] == 0}, {aa, bb, cc}]
```

```
{{cc → 0, aa → 0, bb → 0}}
```

```
(* Quoficients per a la tercera derivada central a 5 punts *)

(* Hipòtesi: la fórmula ha de ser exacta per al polinomi de grau 4 *)
(* Hipòtesi: y'''(i)= aa y(i+2) + bb y(i+1) + cc y(i) + dd y(i-1) + ee y(i-2)   *)

y[x_] = a + b x + c x^2 + d x^3 + e x^4 ;
dc = D[y[x], {x, 3}] /. x → 0;
dcy[x_] := aa y[-2 h] + bb y[- h] + cc y[0] + dd y[h] + ee y[2 h] ;
Solve[{Coefficient[dcy[x] - dc, a] == 0, Coefficient[dcy[x] - dc, b] == 0,
  Coefficient[dcy[x] - dc, c] == 0, Coefficient[dcy[x] - dc, d] == 0,
  Coefficient[dcy[x] - dc, e] == 0}, {aa, bb, cc, dd, ee}]
```

$$\left\{\left\{cc \to 0,\ aa \to -\frac{1}{2\,h^3},\ bb \to \frac{1}{h^3},\ dd \to -\frac{1}{h^3},\ ee \to \frac{1}{2\,h^3}\right\}\right\}$$

```
(* Quoficients per a la tercera derivada central a 7 punts *)

(* Hipòtesi: la fórmula ha de ser exacta per al polinomi de grau 5 *)
(* Hipòtesi: y'''(i)=
 aa y(i+3) + bb y(i+2) + cc y(i+1) + dd y(i) + ee y(i-1) + ff(i-2) + gg(i-3)   *)

y[x_] = a + b x + c x^2 + d x^3 + e x^4 + f x^5 + g x^6 ;
dc = D[y[x], {x, 3}] /. x → 0;
dcy[x_] := aa y[-3 h] + bb y[-2 h] + cc y[-h] + dd y[0] + ee y[h] + ff y[2 h] + gg y[3 h] ;
Solve[{Coefficient[dcy[x] - dc, a] == 0, Coefficient[dcy[x] - dc, b] == 0,
  Coefficient[dcy[x] - dc, c] == 0, Coefficient[dcy[x] - dc, d] == 0,
  Coefficient[dcy[x] - dc, e] == 0, Coefficient[dcy[x] - dc, f] == 0,
  Coefficient[dcy[x] - dc, g] == 0}, {aa, bb, cc, dd, ee, ff, gg}]
```

$$\left\{\left\{dd \to 0,\ aa \to \frac{1}{8\,h^3},\ bb \to -\frac{1}{h^3},\ cc \to \frac{13}{8\,h^3},\ ee \to -\frac{13}{8\,h^3},\ ff \to \frac{1}{h^3},\ gg \to -\frac{1}{8\,h^3}\right\}\right\}$$

```
(* Quoficients per a la tercera derivada central a 9 punts *)

(* Hipòtesi: la fórmula ha de ser exacta per al polinomi de grau 6 *)
(* Hipòtesi: y'''(i)= aa y(i+3) + bb y(i+2) + cc y(i+1) + dd y(i) + ee y(i-1) +
  ff(i-2) + gg(i-3) + ff(i-2) + gg(i-3)+ mm y(i-3) + nn y(i-4)   *)

y[x_] = a + b x + c x^2 + d x^3 + e x^4 + f x^5 + g x^6 + m x^7 + n x^8;
dc = D[y[x], {x, 3}] /. x → 0;
dcy[x_] := aa y[-4 h] + bb y[-3 h] + cc y[-2 h] +
    dd y[-h] + ee y[0] + ff y[h] + gg y[2 h] + mm y[3 h] + nn y[4 h];
Solve[{Coefficient[dcy[x] - dc, a] == 0, Coefficient[dcy[x] - dc, b] == 0,
  Coefficient[dcy[x] - dc, c] == 0, Coefficient[dcy[x] - dc, d] == 0,
  Coefficient[dcy[x] - dc, e] == 0, Coefficient[dcy[x] - dc, f] == 0,
  Coefficient[dcy[x] - dc, g] == 0, Coefficient[dcy[x] - dc, m] == 0,
  Coefficient[dcy[x] - dc, n] == 0}, {aa, bb, cc, dd, ee, ff, gg, mm, nn}]
```

$$\left\{\left\{ee \to 0,\ aa \to -\frac{7}{240\,h^3},\ bb \to \frac{3}{10\,h^3},\ cc \to -\frac{169}{120\,h^3},\right.\right.$$
$$\left.\left.dd \to \frac{61}{30\,h^3},\ ff \to -\frac{61}{30\,h^3},\ gg \to \frac{169}{120\,h^3},\ mm \to -\frac{3}{10\,h^3},\ nn \to \frac{7}{240\,h^3}\right\}\right\}$$

```
(* Quoficients per a la tercera derivada central a 11 punts *)

(* Hipòtesi: la fórmula ha de ser exacta per al polinomi de grau 7 *)
(* Hipòtesi: y'''(i)= aa y(i+5) + bb y(i+4) + cc y(i+3) +
  dd y(i+2) + ee y(i+1)  + ff y(i) + gg y(i-1) + mm y(i-2) + nn y(i-3)
                     + pp y(i-4)+ qq y(i-5)    *)

y[x_] = a + b x + c x^2 + d x^3 + e x^4  + f x^5 + g x^6 + m x^7 + n x^8 + p x^9 + q x^10;
dc = D[y[x], {x, 3}] /. x → 0;
dcy[x_] := aa y[-5 h] + bb y[-4 h] + cc y[-3 h] + dd y[-2 h] +
   ee y[-h] + ff y[0] + gg y[h] + mm y[2 h] + nn y[3 h] + pp y[4 h] + qq y[5 h];
Solve[{Coefficient[dcy[x] - dc, a] == 0, Coefficient[dcy[x] - dc, b] == 0,
  Coefficient[dcy[x] - dc, c] == 0, Coefficient[dcy[x] - dc, d] == 0,
  Coefficient[dcy[x] - dc, e] == 0, Coefficient[dcy[x] - dc, f] == 0,
  Coefficient[dcy[x] - dc, g] == 0, Coefficient[dcy[x] - dc, m] == 0,
  Coefficient[dcy[x] - dc, n] == 0, Coefficient[dcy[x] - dc, p] == 0,
  Coefficient[dcy[x] - dc, q] == 0}, {aa, bb, cc, dd, ee, ff, gg, mm, nn, pp, qq}]
```

$$\left\{\left\{ff \to 0,\ aa \to \frac{41}{6048\ h^3},\ bb \to -\frac{1261}{15\,120\ h^3},\ cc \to \frac{541}{1120\ h^3},\ dd \to -\frac{4369}{2520\ h^3},\ ee \to \frac{1669}{720\ h^3},\right.\right.$$
$$\left.\left.gg \to -\frac{1669}{720\ h^3},\ mm \to \frac{4369}{2520\ h^3},\ nn \to -\frac{541}{1120\ h^3},\ pp \to \frac{1261}{15\,120\ h^3},\ qq \to -\frac{41}{6048\ h^3}\right\}\right\}$$

```
(* Quoficients per a la tercera derivada central a 13 punts *)

y[x_] = a + b x + c x^2 + d x^3 + e x^4  + f x^5 + g x^6 + m x^7 + n x^8 + p x^9 + q x^10 + r x^11 + s x^12;
dc = D[y[x], {x, 3}] /. x → 0;
dcy[x_] := aa y[-6 h] + bb y[-5 h] + cc y[-4 h] + dd y[-3 h] + ee y[-2 h] + ff y[-h] +
   gg y[0] + mm y[h] + nn y[2 h] + pp y[3 h] + qq y[4 h] + rr y[5 h] + ss y[6 h];
Solve[{Coefficient[dcy[x] - dc, a] == 0, Coefficient[dcy[x] - dc, b] == 0,
  Coefficient[dcy[x] - dc, c] == 0, Coefficient[dcy[x] - dc, d] == 0,
  Coefficient[dcy[x] - dc, e] == 0, Coefficient[dcy[x] - dc, f] == 0,
  Coefficient[dcy[x] - dc, g] == 0, Coefficient[dcy[x] - dc, m] == 0,
  Coefficient[dcy[x] - dc, n] == 0, Coefficient[dcy[x] - dc, p] == 0,
  Coefficient[dcy[x] - dc, q] == 0, Coefficient[dcy[x] - dc, r] == 0,
  Coefficient[dcy[x] - dc, s] == 0}, {aa, bb, cc, dd, ee, ff, gg, mm, nn, pp, qq, rr, ss}]
```

$$\left\{\left\{gg \to 0,\ aa \to -\frac{479}{302\,400\ h^3},\ bb \to \frac{19}{840\ h^3},\ cc \to -\frac{643}{4200\ h^3},\right.\right.$$
$$dd \to \frac{4969}{7560\ h^3},\ ee \to -\frac{4469}{2240\ h^3},\ ff \to \frac{1769}{700\ h^3},\ mm \to -\frac{1769}{700\ h^3},\ nn \to \frac{4469}{2240\ h^3},$$
$$\left.\left.pp \to -\frac{4969}{7560\ h^3},\ qq \to \frac{643}{4200\ h^3},\ rr \to -\frac{19}{840\ h^3},\ ss \to \frac{479}{302\,400\ h^3}\right\}\right\}$$

```
(* Quoficients per a la tercera derivada central a 15 punts *)

y[x_] = a + b x + c x^2 + d x^3 + e x^4 + f x^5 +
    g x^6 + m x^7 + n x^8 + p x^9 + q x^10 + r x^11 + s x^12 + t x^13 + u x^14;
dc = D[y[x], {x, 3}] /. x → 0;
dcy[x_] :=
  aa y[-7 h] + bb y[-6 h] + cc y[-5 h] + dd y[-4 h] + ee y[-3 h] + ff y[-2 h] + gg y[-h] +
   mm y[0] + nn y[h] + pp y[2 h] + qq y[3 h] + rr y[4 h] + ss y[5 h] + tt y[6 h] + uu y[7 h];
Solve[{Coefficient[dcy[x] - dc, a] == 0, Coefficient[dcy[x] - dc, b] == 0,
  Coefficient[dcy[x] - dc, c] == 0, Coefficient[dcy[x] - dc, d] == 0,
  Coefficient[dcy[x] - dc, e] == 0, Coefficient[dcy[x] - dc, f] == 0,
  Coefficient[dcy[x] - dc, g] == 0, Coefficient[dcy[x] - dc, m] == 0,
  Coefficient[dcy[x] - dc, n] == 0, Coefficient[dcy[x] - dc, p] == 0,
  Coefficient[dcy[x] - dc, q] == 0, Coefficient[dcy[x] - dc, r] == 0,
  Coefficient[dcy[x] - dc, s] == 0, Coefficient[dcy[x] - dc, t] == 0,
  Coefficient[dcy[x] - dc, u] == 0},
 {aa, bb, cc, dd, ee, ff, gg, mm, nn, pp, qq, rr, ss, tt, uu}]
```

$$\left\{\left\{mm \to 0,\ aa \to \frac{59}{158\,400\,h^3},\ bb \to -\frac{20\,137}{3\,326\,400\,h^3},\ cc \to \frac{2077}{44\,352\,h^3},\ dd \to -\frac{31\,957}{138\,600\,h^3},\right.\right.$$
$$ee \to \frac{247\,081}{302\,400\,h^3},\ ff \to -\frac{222\,581}{100\,800\,h^3},\ gg \to \frac{90\,281}{33\,600\,h^3},\ nn \to -\frac{90\,281}{33\,600\,h^3},\ pp \to \frac{222\,581}{100\,800\,h^3},$$
$$\left.\left.qq \to -\frac{247\,081}{302\,400\,h^3},\ rr \to \frac{31\,957}{138\,600\,h^3},\ ss \to -\frac{2077}{44\,352\,h^3},\ tt \to \frac{20\,137}{3\,326\,400\,h^3},\ uu \to -\frac{59}{158\,400\,h^3}\right\}\right\}$$

```
(* Quoficients per a la tercera derivada central a 17 punts *)

y[x_] = a + b x + c x^2 + d x^3 + e x^4 + f x^5 + g x^6 +
    m x^7 + n x^8 + p x^9 + q x^10 + r x^11 + s x^12 + t x^13 + u x^14 + v x^15 + z x^16;
dc = D[y[x], {x, 3}] /. x → 0;
dcy[x_] := aa y[-8 h] + bb y[-7 h] + cc y[-6 h] + dd y[-5 h] +
    ee y[-4 h] + ff y[-3 h] + gg y[-2 h] + mm y[-h] + nn y[0] + pp y[h] +
    qq y[2 h] + rr y[3 h] + ss y[4 h] + tt y[5 h] + uu y[6 h] + vv y[7 h] + zz y[8 h];
Solve[{Coefficient[dcy[x] - dc, a] == 0, Coefficient[dcy[x] - dc, b] == 0,
  Coefficient[dcy[x] - dc, c] == 0,
  Coefficient[dcy[x] - dc, d] == 0, Coefficient[dcy[x] - dc, e] == 0,
  Coefficient[dcy[x] - dc, f] == 0, Coefficient[dcy[x] - dc, g] == 0,
  Coefficient[dcy[x] - dc, m] == 0, Coefficient[dcy[x] - dc, n] == 0,
  Coefficient[dcy[x] - dc, p] == 0, Coefficient[dcy[x] - dc, q] == 0,
  Coefficient[dcy[x] - dc, r] == 0, Coefficient[dcy[x] - dc, s] == 0,
  Coefficient[dcy[x] - dc, t] == 0, Coefficient[dcy[x] - dc, u] == 0,
  Coefficient[dcy[x] - dc, v] == 0, Coefficient[dcy[x] - dc, z] == 0},
 {aa, bb, cc, dd, ee, ff, gg, mm, nn, pp, qq, rr, ss, tt, uu, vv, zz}]
```

$$\left\{\left\{nn \to 0,\ aa \to -\frac{266\,681}{3\,027\,024\,000\,h^3},\ bb \to \frac{21\,701}{13\,513\,500\,h^3},\ cc \to -\frac{1\,058\,149}{75\,675\,600\,h^3},\ dd \to \frac{41\,981}{540\,540\,h^3},\right.\right.$$
$$ee \to -\frac{1\,033\,649}{3\,326\,400\,h^3},\ ff \to \frac{999\,349}{1\,039\,500\,h^3},\ gg \to -\frac{901\,349}{378\,000\,h^3},\ mm \to \frac{372\,149}{132\,300\,h^3},$$
$$pp \to -\frac{372\,149}{132\,300\,h^3},\ qq \to \frac{901\,349}{378\,000\,h^3},\ rr \to -\frac{999\,349}{1\,039\,500\,h^3},\ ss \to \frac{1\,033\,649}{3\,326\,400\,h^3},$$
$$\left.\left.tt \to -\frac{41\,981}{540\,540\,h^3},\ uu \to \frac{1\,058\,149}{75\,675\,600\,h^3},\ vv \to -\frac{21\,701}{13\,513\,500\,h^3},\ zz \to \frac{266\,681}{3\,027\,024\,000\,h^3}\right\}\right\}$$

```
(* Quoficients per a la tercera derivada central a 19 punts *)

y[x_] = a + b x + c x^2 + d x^3 + e x^4 + f x^5 + g x^6 + m x^7 + n x^8 +
    p x^9 + q x^10 + r x^11 + s x^12 + t x^13 + u x^14 + v x^15 + z x^16 + j x^17 + k x^18;
dc = D[y[x], {x, 3}] /. x → 0;
dcy[x_] := aa y[-9 h] + bb y[-8 h] + cc y[-7 h] + dd y[-6 h] + ee y[-5 h] +
    ff y[-4 h] + gg y[-3 h] + mm y[-2 h] + nn y[-h] + pp y[0] + qq y[h] + rr y[2 h] +
    ss y[3 h] + tt y[4 h] + uu y[5 h] + vv y[6 h] + zz y[7 h] + jj y[8 h] + kk y[9 h];
Solve[{Coefficient[dcy[x] - dc, a] == 0, Coefficient[dcy[x] - dc, b] == 0,
  Coefficient[dcy[x] - dc, c] == 0,
  Coefficient[dcy[x] - dc, d] == 0, Coefficient[dcy[x] - dc, e] == 0,
  Coefficient[dcy[x] - dc, f] == 0, Coefficient[dcy[x] - dc, g] == 0,
  Coefficient[dcy[x] - dc, m] == 0, Coefficient[dcy[x] - dc, n] == 0,
  Coefficient[dcy[x] - dc, p] == 0, Coefficient[dcy[x] - dc, q] == 0,
  Coefficient[dcy[x] - dc, r] == 0, Coefficient[dcy[x] - dc, s] == 0,
  Coefficient[dcy[x] - dc, t] == 0, Coefficient[dcy[x] - dc, u] == 0,
  Coefficient[dcy[x] - dc, v] == 0, Coefficient[dcy[x] - dc, z] == 0,
  Coefficient[dcy[x] - dc, j] == 0, Coefficient[dcy[x] - dc, k] == 0},
 {aa, bb, cc, dd, ee, ff, gg, mm, nn, pp, qq, rr, ss, tt, uu, vv, zz, jj, kk}]
```

$$\left\{\left\{pp \to 0,\ aa \to \frac{63\,397}{3\,027\,024\,000\,h^3},\ bb \to -\frac{10\,949}{25\,872\,000\,h^3},\ cc \to \frac{196\,909}{48\,048\,000\,h^3},\ dd \to -\frac{9\,601\,741}{378\,378\,000\,h^3},\right.\right.$$
$$ee \to \frac{5861}{51\,744\,h^3},\ ff \to -\frac{9\,381\,241}{24\,024\,000\,h^3},\ gg \to \frac{9\,072\,541}{8\,316\,000\,h^3},\ mm \to -\frac{8\,190\,541}{3\,234\,000\,h^3},\ nn \to \frac{3\,427\,741}{1\,176\,000\,h^3},$$
$$qq \to -\frac{3\,427\,741}{1\,176\,000\,h^3},\ rr \to \frac{8\,190\,541}{3\,234\,000\,h^3},\ ss \to -\frac{9\,072\,541}{8\,316\,000\,h^3},\ tt \to \frac{9\,381\,241}{24\,024\,000\,h^3},\ uu \to -\frac{5861}{51\,744\,h^3},$$
$$\left.\left.vv \to \frac{9\,601\,741}{378\,378\,000\,h^3},\ zz \to -\frac{196\,909}{48\,048\,000\,h^3},\ jj \to \frac{10\,949}{25\,872\,000\,h^3},\ kk \to -\frac{63\,397}{3\,027\,024\,000\,h^3}\right\}\right\}$$

```
(* Quoficients per a la tercera derivada central a 21 punts *)

y[x_] = a + b x + c x^2 + d x^3 + e x^4 + f x^5 + g x^6 + m x^7 + n x^8 + p x^9 + q x^10 +
    r x^11 + s x^12 + t x^13 + u x^14 + v x^15 + z x^16 + j x^17 + k x^18 + ab x^19 + ac x^20;
dc = D[y[x], {x, 3}] /. x → 0;
dcy[x_] := aa y[-10 h] + bb y[-9 h] + cc y[-8 h] + dd y[-7 h] + ee y[-6 h] + ff y[-5 h] +
    gg y[-4 h] + mm y[-3 h] + nn y[-2 h] + pp y[-h] + qq y[0] + rr y[h] + ss y[2 h] + tt y[3 h] +
    uu y[4 h] + vv y[5 h] + zz y[6 h] + jj y[7 h] + kk y[8 h] + abab y[9 h] + acac y[10 h];
Solve[{Coefficient[dcy[x] - dc, a] == 0, Coefficient[dcy[x] - dc, b] == 0,
  Coefficient[dcy[x] - dc, c] == 0, Coefficient[dcy[x] - dc, d] == 0,
  Coefficient[dcy[x] - dc, e] == 0, Coefficient[dcy[x] - dc, f] == 0,
  Coefficient[dcy[x] - dc, g] == 0, Coefficient[dcy[x] - dc, m] == 0,
  Coefficient[dcy[x] - dc, n] == 0, Coefficient[dcy[x] - dc, p] == 0,
  Coefficient[dcy[x] - dc, q] == 0, Coefficient[dcy[x] - dc, r] == 0,
  Coefficient[dcy[x] - dc, s] == 0, Coefficient[dcy[x] - dc, t] == 0,
  Coefficient[dcy[x] - dc, u] == 0, Coefficient[dcy[x] - dc, v] == 0,
  Coefficient[dcy[x] - dc, z] == 0, Coefficient[dcy[x] - dc, j] == 0,
  Coefficient[dcy[x] - dc, k] == 0, Coefficient[dcy[x] - dc, ab] == 0,
  Coefficient[dcy[x] - dc, ac] == 0},
 {aa, bb, cc, dd, ee, ff, gg, mm, nn, pp, qq, rr, ss, tt, uu, vv, zz, jj, kk, abab, acac}]
```

$$\left\{\left\{qq \to 0,\ aa \to -\frac{514\,639}{102\,918\,816\,000\,h^3},\ bb \to \frac{11\,419}{102\,918\,816\,h^3},\ cc \to -\frac{487\,121}{411\,675\,264\,h^3},\right.\right.$$
$$dd \to \frac{5663}{700\,128\,h^3},\ ee \to -\frac{1\,933\,049}{48\,432\,384\,h^3},\ ff \to \frac{9\,587\,629}{63\,063\,000\,h^3},\ gg \to -\frac{1\,888\,949}{4\,036\,032\,h^3},$$
$$mm \to \frac{1\,827\,209}{1\,513\,512\,h^3},\ nn \to -\frac{1\,650\,809}{620\,928\,h^3},\ pp \to \frac{698\,249}{232\,848\,h^3},\ rr \to -\frac{698\,249}{232\,848\,h^3},\ ss \to \frac{1\,650\,809}{620\,928\,h^3},$$
$$tt \to -\frac{1\,827\,209}{1\,513\,512\,h^3},\ uu \to \frac{1\,888\,949}{4\,036\,032\,h^3},\ vv \to -\frac{9\,587\,629}{63\,063\,000\,h^3},\ zz \to \frac{1\,933\,049}{48\,432\,384\,h^3},$$
$$\left.\left.jj \to -\frac{5663}{700\,128\,h^3},\ kk \to \frac{487\,121}{411\,675\,264\,h^3},\ abab \to -\frac{11\,419}{102\,918\,816\,h^3},\ acac \to \frac{514\,639}{102\,918\,816\,000\,h^3}\right\}\right\}$$